Chapter 2: Algebra

Open Sentences

An **open sentence** is a mathematics sentence with something missing. Look at this open sentence:

$$10 + \square = 35$$

A number is missing from this number sentence. Since you know this is an addition sentence, you can use subtraction to find the missing number: 35 – 10 = 25. This strategy can also work for open sentences where the operation is subtraction, multiplication, or division.

$\square - 4 = 98 \longrightarrow 98 + 4 = \boxed{102}$

$7 \times \square = 35 \longrightarrow 35 \div 7 = \boxed{5}$

$\square \div 9 = 9 \longrightarrow 9 \times 9 = \boxed{81}$

In some division sentences you do not have to use inverse operations to solve for a missing number. If the missing number is being used to divide another number, you can simply divide the larger number by the value that is given. The open sentence 10 ÷ ____ = 5, for example, can be rewritten as 10 ÷ 5 = ____ to find the missing value of 2.

Some open sentences use letters in place of blank boxes or lines. To write the open sentences above with letters, you might write:

$10 + x = 35$ $\quad$ $y - 4 = 98$ $\quad$ $7 \times m = 35$ $\quad$ $a \div 9 = 9$

Just think of the letter as an empty box or a blank line. Then solve.

1 $60 \div b = 5$

What is the value of b?

A 6

B 12

C 55

D 300

Think it through: Because the missing number here is being used to divide another number, you do not need to multiply to solve for b. Instead, write this as a division sentence with b to the right of the equals sign: $60 \div 5 = b$. Then solve; the value of b is 12. **The correct answer is B.**

On Your Own

2 $16 + n = 25 - 2$

What is the value of n?

F 7 $\quad$ **H** 18

G 9 $\quad$ **J** 23

Missing Operations

Sometimes you must find an operation that is missing from a number sentence. Look at this example:

$$100 \square 5 = 20$$

This sentence is missing the symbol that tells whether to multiply, divide, add, or subtract. But which is it? Look closely at the numbers. 20 is a much smaller whole number than 100, so you know this problem will use an operation to make 100 smaller, either subtraction or division. Try subtraction: 100 – 5 = 95. 95 is much bigger than 20, so the missing operation is not subtraction; it is more likely to be division. To check whether you are correct, write 100 ÷ 5 this way and solve:

$$5\overline{)100}\quad 20$$

So the missing operation is definitely division. You can use this strategy whenever you must find the operation that is missing from a number sentence.

1 $361 \square 49 = 410$

What symbol belongs in the box?

A +

B –

C ×

D ÷

Think it through: Look at the numbers and compare. You can see that the missing operation will make the whole number 361 bigger. So the missing operation is either multiplication or addition. At this point you might estimate to decide between the two operations; round 361 to 360 and 49 to 50, then add: 360 + 50 = 410. So you know the missing operation is addition. **The correct answer is A.**

On Your Own

2 $\frac{3}{4} \square \frac{1}{4} = 1 - \frac{1}{2}$

What is the missing symbol?

F +

G –

H ×

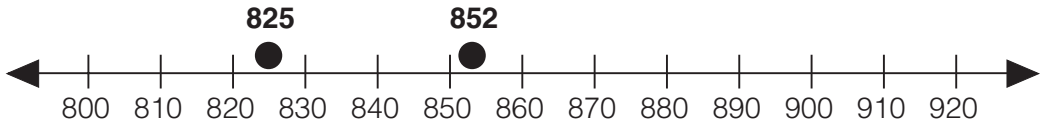
J ÷

Using Symbols to Compare Numbers

You can use symbols to compare whole numbers, fractions, and decimals.

$>$	*greater than*
$<$	*less than*
$=$	*equal to*
$\neq$	*not equal to*

Compare 825 and 852. First find them on this number line.



You can see that all these number sentences are true:

852 $>$ 825, or 852 *is greater than 825*

825 $<$ 852, or 825 *is less than 852*

852 $\neq$ 825, or 852 *is not equal to 825*

1 0.50 $\square$ $\frac{1}{2}$

What symbol belongs in the box?

A +

B <

C =

D ≠

Think it through: Here you have 2 different ways of expressing numbers, a decimal and a fraction. When you have to compare a decimal and a fraction, try to convert them both to the same kind of number. Let's change the decimal to a fraction. You know that .5 = five tenths or $\frac{5}{10}$. In its simplest, reduced form, $\frac{5}{10} = \frac{1}{2}$, so the two numbers are equal. **The correct answer is C.**

On Your Own

2 $5 \times 3 < \square > 20 - 2$

Which number belongs in the box?

F 15

G 16

H 18

J 19

Number Patterns

You can make number patterns in many ways. You could describe the **rule** for the pattern below as *add 2*:

12, 14, 16, 18, 20 . . .

Another pattern shows numbers become smaller by $\frac{2}{3}$. You could describe the rule for this pattern as *subtract* $\frac{2}{3}$:

15, $14\frac{1}{3}$, $13\frac{2}{3}$, 13, $12\frac{1}{3}$ . . .

And here, each number is 3 times the number before it. You could describe the rule as *multiply by 3*:

2, 6, 18, 54, 162 . . .

To figure out the rule for a pattern, compare the first two numbers. Make a good guess as to whether the rule uses addition, subtraction, multiplication, or division. Then check your guess using other numbers in the pattern. Remember, if the numbers in a pattern become larger, the rule uses addition or multiplication. If the numbers become smaller, it uses subtraction or division. Sometimes you have to use more than one operation.

1 What is the rule for this pattern?
\$2.25, \$2.19, \$2.13, \$2.07, \$2.01

A multiply by 6

B add 0.6

C divide by 6

D subtract 0.06

Think it through: Look at the first 2 amounts in the pattern; you can see they get smaller. So the rule uses subtraction or division. However, you can also see that the amounts decrease by just a few cents each time, so you can guess that the rule uses subtraction and not division. To check, try subtracting \$2.19 from \$2.25. The difference is 6¢. So try using a "subtract 6¢" rule on the next amount, \$2.19. If you subtract 6¢ from \$2.19, you get \$2.13, which is the third amount in the pattern. So "subtract .06" or "subtract 6¢" is the rule. **The correct answer is D.**

On Your Own

2 What is the next number in this pattern?
128, 64, 32, 16, 8, ___

F 64

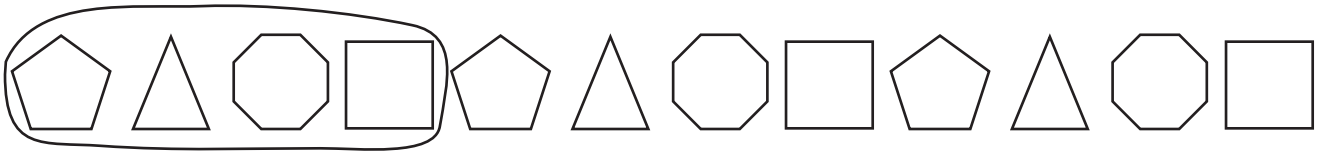
G 4

H 2

J $\frac{1}{2}$

Shape Patterns

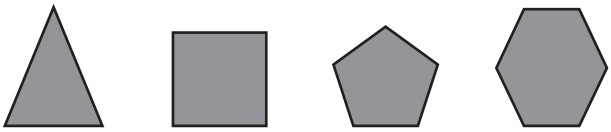
A pattern is made up of a **unit** that repeats. Some patterns are made up of shapes. Look at these shapes:



1 unit

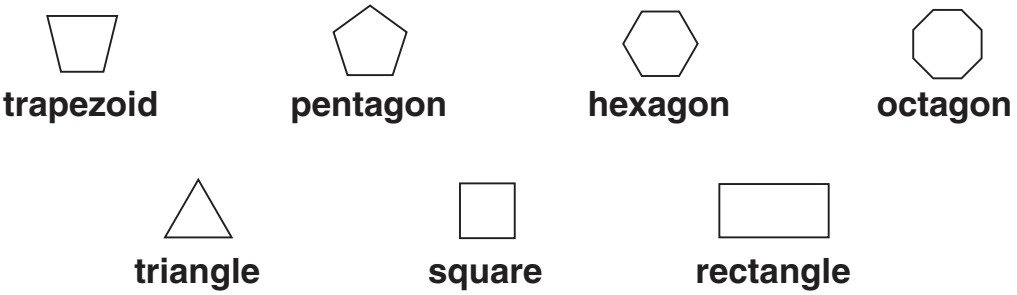
You can describe the pattern above this way: *pentagon*, *triangle*, *octagon*, *square*. This is the pattern unit that repeats.

In some patterns, shapes do not repeat but rather change in another way. To figure out the rule, look closely at the shapes. For example:



Although these shapes are all different, you can see that each shape has one more side than the one before it.

Tip: Remember the names for different shapes.



1 Marcus made this pattern with shape tiles. How can he describe his pattern?

A octagon, pentagon, rectangle, pentagon

B rectangle, triangle, rectangle, octagon

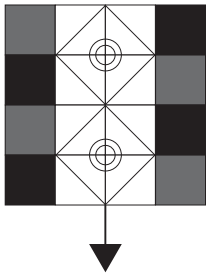
C pentagon, octagon, rectangle, octagon

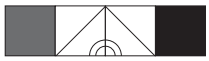
D pentagon, triangle, rectangle, octagon

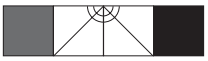
Think it through: Look carefully at Marcus's pattern; the first shape is a pentagon. Therefore the correct answer choice must include a pentagon in it. The next shape is an octagon, so the correct choice must include a pentagon and octagon, in that order. The only choice that includes this order of shapes is C. To check, compare choice C to the pattern; yes, there is a sequence of pentagon, octagon, rectangle, and octagon in Marcus's pattern. **The correct answer is C.**

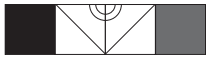
On Your Own

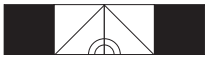
2 Alisa drew this pattern. Which choice shows what Alisa should draw next to continue her pattern?



F 

G 

H 

J 

3 Elliot is helping his mother replace some missing tiles in their bathroom. Which 2 shapes does he need to put in the blanks to continue the pattern?

A circle, circle

B circle, pentagon

C triangle, square

D pentagon, triangle

Number Sentences and Word Problems

Most mathematics tests include word problems for you to solve. To solve a word problem:

- Read carefully. Pay attention to the numbers in the problem.
- Look for word clues that tell whether you should add, subtract, multiply, or divide. Sometimes you may have to use more than one operation to solve a problem.
- Remember that some problems include extra information. Circle this information so you don't become confused.
- Write a number sentence about the problem. Solve.
- If you have time, check your answer using estimation or inverse operations.

Tip: Write your number sentence so the missing number comes after the equals sign. For example, 235 – 129 = ____ is easier to solve than 235 – ____ = 129, even though they both mean the same thing.

1 Myla's soccer team, the Inwood All-Stars, is getting in shape for a new season. The first Monday of practice they will run for 10 minutes. On Tuesday they will run for 15 minutes. Wednesday, they will run for 20 minutes. They will increase their running time each day until they can run for 35 minutes.

On what day will Myla's soccer team run for half an hour?

Answer _____

Think it through: Read the problem carefully. Remember that half an hour is the same as 30 minutes. Now look at the other numbers. You can see that they make a pattern of "plus 5": 10 minutes, 15 minutes, and so on. To find out what day it will be when the team runs for 30 minutes, subtract the number of minutes they ran on the third day, Wednesday, from 30 minutes. Then divide the difference by 5 to find out how many days to "skip ahead" from Wednesday. So your number sentence might look like this: (30 – 20) ÷ 5. If you solve this sentence, you get an answer of 2, so Myla's team will run 30 minutes 2 days from Wednesday, which is Friday.

To check your answer, you might skip count by 5s from Wednesday: 25 minutes on Thursday, 30 minutes on Friday. You may also create a chart like the one below. Now you know that the correct answer is **Friday**.

M	T	W	Th	F
10	15	20	25	30

On Your Own

2 Duncan is 6 years younger than his brother Gerard. Gerard is 3 times as old as their sister Lisa. Lisa is 1 year older than her friend Kate. If Lisa is 5 years old, how old is Duncan?

Answer _____

Algebra
Mini-Test

1 What number belongs in the box?

$40 \div \square = 5$

A 8

B 35

C 45

D 200

2 What symbol belongs in the box?

$33 \square 11 = 363$

F +

G –

H ×

J ÷

3 Jacob saw this pattern of tiles on a mosaic in the Metropolitan Museum of Art. How can he describe it in a postcard to his grandmother?

A square, hexagon, square, pentagon, circle

B hexagon, circle, square, hexagon, rectangle

C square, pentagon, square, hexagon, circle

D circle, hexagon, square, pentagon, square

4 Which number below makes this sentence true?

$4 \times 3 < \_\_\_\_\_ < 20 - 2$

F 11

G 12

H 14

J 18

5 What number comes next in the pattern?
$1.82, $1.88, $1.92, $1.98, _____

A $1.99

B $1.85

C $2.02

D $2.12

6 Lucy has 12 blue marbles and *n* red marbles. James has 21 marbles in all. If Lucy has 3 fewer marbles than James, how many red marbles does Lucy have?

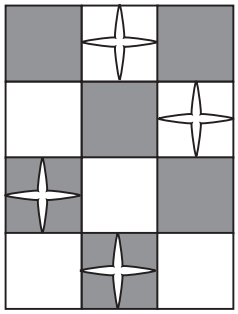
F 3

G 6

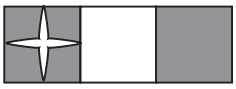
H 12

J 21

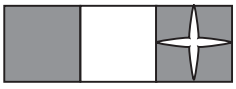
7 Jillian drew this pattern for art class:

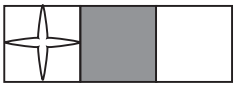


Which choice shows what Jillian should draw next to continue her pattern?

A 

B 

C 

D 

Fun with Algebra

1 Frozen Fun

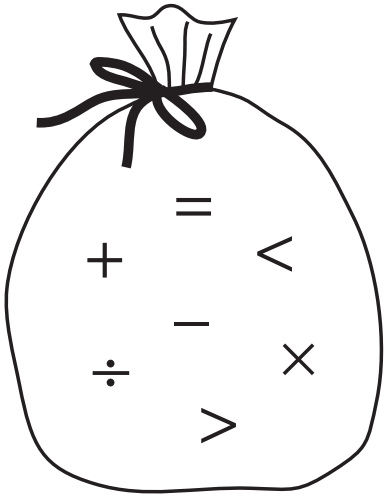
Blizzard Brian is so cold that his mathematics problems are frozen! Help Blizzard Brian thaw out the value of each snowflake.

$4 \times$ ❄ $= 36$	❄ $+ 3 = 17$	$18 -$ ❄ $= 1$	$28 \div$ ❄ $= 7$
❄ $=$ _____	❄ $=$ _____	❄ $=$ _____	❄ $=$ _____

2 Symbol Snatcher

The Symbol Snatcher strikes again! Use the symbols the police found in the symbol snatcher's sack to make the mathematics sentences true.

$\frac{7}{8}$ ____ $\frac{1}{3} + \frac{2}{3}$
$3 \times 4 = 15$ ____ 3
$19 - 4$ ____ 14
2 ____ $2 = 16$ ____ 4
$\frac{2}{10}$ ____ $\frac{1}{5}$



3 Cryptic Carvings

King Tutenmathrocks was carving a pattern of hieroglyphics on his pyramid but never finished. What symbols are next in King Tutenmathrocks's pattern?

 _____ _____